

**考试科目： 概率论与数理统计 开课单位： 数学系**

**考试时长： ２小时 命题教师：概率论与数理统计教学组**

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| 题号 | Part 1 | Part 2 | Part 3 | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 分值 |  |  |  |  |  |  |  |  |

本试卷共三大部分，满分100分（考试结束后请将试卷、答题本、草稿纸一起交给监考老师）

**第一部分 选择题（每题4分，总共20分）**

**Part One – Single Choice (4 marks each question, 20 marks in total)**

1. 设A，B，C表示三个事件，则 ( ).

(A) A，B，C中有一个发生;

(B) A，B，C中恰有两个发生;

(C) A，B，C中不多于一个发生;

(D) A，B，C都不发生.

Assume A，B，C are three events, then means that ( ) .

(A) one of the events A, B, C happens;

(B) two of the events A, B, C happen;

(C) no more than one of the events A, B, C happen;

(D) none of the events A, B, C happens.

2. 甲、乙、丙 3 人独立地译出一种密码，他们能译出的概率分别为1/5,1/3,1/4，则能译出这种密码的概率为( ).

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

There are three people who are independently guessing a password. The probability of individually getting the password is 1/5, 1/3, 1/4 respectively. The probability of getting the password is ( ).

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

3. 设随机变量，，且与相互独立，则服从( )分布.

(A) (B) (C) (D)

Assume random variables ，, and they are independent to each other. Then, follows the distribution of ( ).

(A) (B) (C) (D)

4. 设随机变量和独立同分布，且的分布函数为，则的分布函数为( ).

(A) (B) (C) (D)

Assume that random variables and independent and identically distributed. The distribution of r.v. is . The distribution function of is ( ).

(A) (B) (C) (D)

5. 已知随机变量的分布函数为，且，则( )

(A) 存在点，使得成立;

1. 存在点，使得成立;

(C) 对任意有;

(D) 对任意有.

Assume the distribution of r.v. is , and . Which statement is true? ( )

(A) There is a point of , which makes

(B) There is a point , which makes ;

(C) Any point of makes;

(D) Any point of makes.

**第二部分 填空题（每空2分，总共20分）**

**Part Two – Blank Filling (2 marks each blank, 20 marks in total)**

1. 已知事件相互独立, 事件与互不相容, 且, 设为事件中至少有一个发生, 则\_\_\_\_\_\_\_\_\_\_.

Events are independent to each other. Events and , Events and are disjoint events. Furthermore Assume is the event that at least one of the three Events happens, thus \_\_\_\_\_\_\_\_\_.

1. 已知两个事件满足条件, 且, 则\_\_\_\_\_\_\_\_\_\_.

There are two events , and they have. If , then \_\_\_\_\_\_\_\_\_\_.

1. 设两个相互独立的事件和都不发生的概率为，发生不发生的概率与发生不发生的概率相等，则\_\_\_\_\_\_\_\_\_\_.

Assume there are two independent events and . The probability that both of them don’t happen is . The probability of happens and doesn’t is the same as the probability of happens and doesn’t. Then \_\_\_\_\_\_\_\_\_\_.

1. 设都是一元分布函数, 常数若也是分布函数, 则常数应满足的条件是\_\_\_\_\_\_\_\_.

Assume that are one dimensional distributions with the constants If is a distribution, then the constants satisfy the condition of \_\_\_\_\_\_\_\_.

1. 设随机变量, 令*,* 则随机变量服从的分布及参数为\_\_\_\_\_\_\_\_\_\_.

Assume the random variable , and Then, the distribution of r.v. is \_\_\_\_\_\_\_\_\_\_.

1. 设随机变量服从参数为1的指数分布, 为常数且大于0，则\_\_\_\_\_\_\_\_\_\_.

Assume the random variable follows exponential distribution with the parameter 1. If is a constant being greater than 0, then \_\_\_\_\_\_\_\_\_\_.

1. 设两个相互独立的随机变量与分别服从正态分布和, 则\_\_\_\_\_\_\_\_\_\_.

Assume there are two independent random variablesand , they follow normal distribution and respectively. Then \_\_\_\_\_\_\_\_\_\_.

1. 设随机变量的联合密度函数为, 则中至少有一个小于1/2的概率为\_\_\_\_\_\_\_\_\_\_.

Assume that the joint density function of random variables is

. Then the probability that at least one of the events and happens is\_\_\_\_\_\_\_\_\_\_.

1. 设随机变量与相互独立, 且与均服从区间上的均匀分布, 则\_\_\_\_\_\_\_\_\_\_.

Assume there are two independent random variables , they all follow uniform distribution U Then \_\_\_\_\_\_\_\_\_\_.

1. 设与是两个随机变量, 且, . 则\_\_\_\_\_\_\_\_\_\_.

Assume that and are two random variables, and , . Then \_\_\_\_\_\_\_\_\_\_.

**第三部分 大题（每题10分，总共60分）**

**Part Three – Question Answering (10 marks each question, 60 marks in total)**

1. 学生在做一道有4个选项的单项选择题时, 如果学生不知道正确答案, 就作随机猜测. 现从卷面上看题是答对了, 试在以下情况下求学生确实知道正确答案的概率.
2. 学生知道正确答案和胡乱猜测的概率都是0.5.
3. 学生知道正确答案的概率都是0.2.

A student needs to select 1 choice from 4 optional choices for answering a question. The student will randomly select a choice if having not got the answer, and of course will select it if having obtained the answer. Now the student’s selected choice is the answer, what is the probability that the student has obtained the answer before selecting a choice based on the following scenarios?

1. The probability of having obtained the answer before selecting a choice is 0.5.
2. The probability of having obtained the answer before selecting a choice is 0.2.
3. 设随机变量的概率分布. 在给定的条件下，随机变量服从均匀分布 求的分布函数和密度函数.

Assume the random variable has Under the condition , the random variable follows uniform distribution What are the distribution function and the density function

1. 若每只母鸡产蛋的个数服从参数为的泊松分布, 而每个蛋能孵化成小鸡的概率为. 试证：每只母鸡有只小鸡的概率服从参数为的泊松分布.

If a hen lays eggs which follows Poisson distribution with the parameter . The probability that each egg transforms into a chick is . Determine the probability of each hen has chicks () such that follows Poisson distribution with the parameter .

4．设，其中随机变量的密度函数为

1. 求常数.
2. 求的密度函数.

Assume ，and the density function of r.v. is:

1. What is the constant ?
2. What is the density function
3. 已知随机变量和的分布函数分别为

且已知，求：

1. 和的联合频率函数；
2. 和是否独立？
3. 时，的条件频率函数.

Assume the distribution functions of r.v. and r.v. are as follows.

Furthermore, .

1. What is the joint frequency function of r.v. and r.v. ?
2. Are and independent？
3. When ，what is the conditional probability?
4. 设二维随机变量的联合密度函数为

1. 确定常数;
2. 求边际密度函数;
3. 求函数的分布函数.

Assume the joint density function of the two-dimensional random variable

1. Compute the constant ;
2. Find the marginal density function of ;
3. Find the distribution function of .

**答案**

**第一部分 选择题（每题4分，总共20分）**

D C C A D

**第二部分 填空题（每空2分，总共20分）**

1. 0.9
2. 2/3
3. 0.5
4. 5/8
5. 1/9
6. 5/7

**第三部分 大题（每题10分，总共60分）**

1. 学生在做一道有四个选项的单项选择题时，如果他不知道正确答案，就作随机猜测。现从卷面上看题是答对了，试在以下情况下求学生确实知道正确答案的概率。
2. 学生知道正确答案和胡乱猜测的概率都是1/2.
3. 学生知道正确答案的概率都是0.2.

解：记事件为“题目答对了”，事件为“知道正确答案”，

则

(1)此时 由贝叶斯公式有

1. 此时 由贝叶斯公式有
2. 设随机变量的概率分布的条件下，随机变量服从均匀分布，求

解：

当时，

当时，

当时，

当时，

1. 若每只母鸡产蛋的个数服从参数为的泊松分布，而每个蛋能孵化成小鸡的概率为.试证：每只母鸡有只小鸡的概率服从参数为的泊松分布。

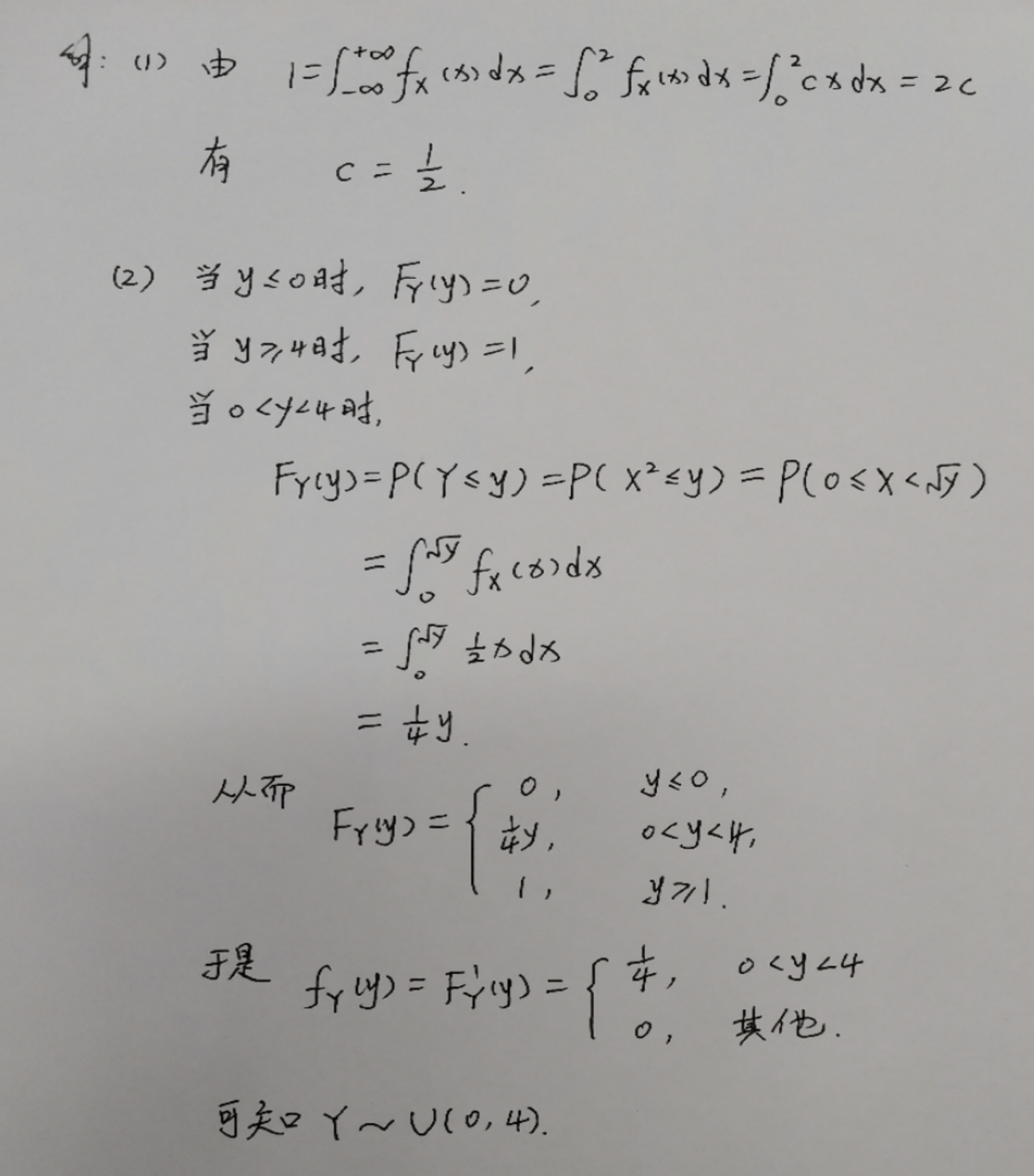
证明：设，由全概率公式，对于任意的正整数有



则服从参数是的泊松分布。

1. 设，其中随机变量的密度函数为

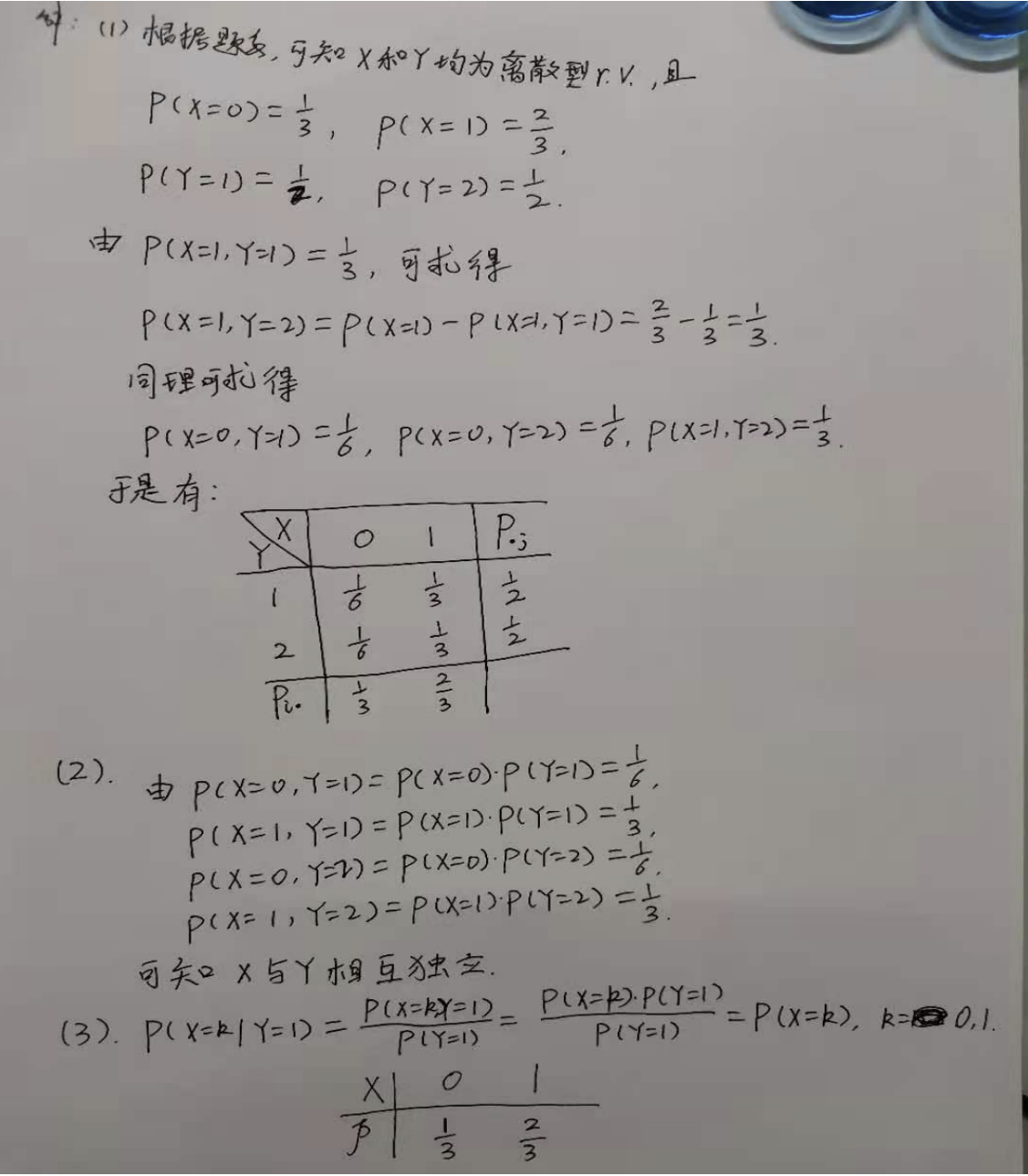
1. 求常数；
2. 求的密度函数。



1. 已知随机变量X和Y的分布函数分别为

且已知，求：

1. X和Y的联合频率函数；
2. X和是否独立？
3. Y=1时，X的条件频率函数。



1. 设二维随机变量的联合密度函数为

1. 确定常数

(2) 求边际密度函数

(3) 求函数的分布函数.

